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Mach bands as empirically derived associations

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ABSTRACT **If Mach bands arise as an empirical consequence of real-world luminance profiles, several predictions follow. First, the appearance of Mach bands should accord with the appearance of naturally occurring highlights and lowlights. Second, altering the slope of an ambiguous luminance gradient so that it corresponds more closely to gradients that are typically adorned with luminance maxima and minima in the position of Mach bands should enhance the illusion. Third, altering a luminance gradient so that it corresponds more closely to gradients that normally lack luminance maxima and minima in the position of Mach bands should diminish the salience of the illusion. Fourth, the perception of Mach bands elicited by the same luminance gradient should be changed by contextual cues that indicate whether the gradient is more or less likely to signify a curved or a flat surface. Because each of these predictions is met, we conclude that Mach bands arise because the association elicited by the stimulus (the percept) incorporates these features as a result of past experience.**

Whereas the specific stimulus used to elicit the Mach band illusion occurs only rarely outside the laboratory, similar luminance gradients generated by curved surfaces typically have photometric highlights and lowlights in the position of the illusory bands (1). Thus, in considering how luminance gradients induce illusory bands of lightness and darkness, Mach and others failed to notice that the perceptual profile of the bands is remarkably similar to the overall luminance profile of curved surfaces, which are typically adorned by photometric maxima and minima. This similarity raises the possibility that luminance gradients induce the perception of Mach bands because the visual system associates these stimuli with the photometric highlights and lowlights that often adorn them.

If this idea is correct, then several predictions follow: (*i*) a depiction of the natural source of the Mach band luminance profile (i.e., a linear gradient preceding an attached shadow) should elicit Mach bands in the position that highlights and lowlights normally occupy; (*ii*) the salience of illusory highlights and lowlights should be enhanced when the stimulus is made more like gradients normally adorned by highlights and lowlights; (*iii*) the salience of the illusion should be diminished when the stimulus is made more like gradients that typically lack highlights and lowlights; and (*iv*) the salience of Mach bands in response to a given luminance gradient should be changed by ancillary information that indicates whether the gradient pertains to a curved or a flat surface.

The present study examines each of these predictions in turn.

METHODS

Computer Graphics. The graphical methods used here are the same as those described in the companion paper (1).

Generation of Luminance Gradients with Different Characteristics. The penumbral gradients presented in Fig. 2 were generated by illuminating one edge of an opaque card with diffuse light from an extended source (a 75 W halogen lamp) illuminating a white matte cardboard surface. The shape of the light source was controlled by a square occluder placed directly over the lamp, which also was fitted with a white plastic diffuser; the square aperture could be rotated through 90°. The images of these gradients were taken with a Polaroid PDC-2000 digital camera and analyzed as previously described (1).

Subjects. The subjects, whose responses are reported in Table 1, were faculty, students, or staff in the Department of Neurobiology at Duke University. All had normal vision, were naive about the purposes of the test or the study more generally, and volunteered their time (~10 min to have the task explained to them and to report their perceptions; see legend of Table 1 for details).

RESULTS

Mach Bands Appear in the Position of Highlights and Lowlights on Computer-Rendered Objects that Lack These Adornments. Fig. 1 compares a digital photograph of a curved real-world surface with a computer-generated version of the same object. Whereas the luminance gradient across the curved edge of the cube in the photograph in Fig. 1*A* is adorned by a photometrically measurable highlight and lowlight, these features have been omitted on the gradient across the depicted cube in Fig. 1*B*. Brightness maxima and minima are nonetheless apparent at the initiation and termination of the luminance gradient across the curved surface of the rendered cube. Although the perceived intensity of the illusory bands is less than the intensity of the real highlights and lowlights, the brighter Mach band in Fig. 1*B* falls in the position of the highlight in Fig. 1*A*, and the dark Mach band falls in the position of the lowlight.

The Perception of Mach Bands is Enhanced by Luminance Gradients Typically Adorned with Highlights and Lowlights and Is Diminished by Those that Lack Them. The implication of the preceding section is that Mach bands arise because illusory brightness maxima and minima are generated by the visual system in the presence of unusual stimuli that share the features of luminance gradients that are typically adorned by these bands as photometric realities (e.g., Mach's spinning disk, or the linear penumbral gradient in Fig. 3 of ref. 1). If this interpretation is correct, then observers should experience a systematic change in the salience of Mach bands as the luminance profile of a stimulus that normally occurs with highlights and lowlights (e.g., a profile generated by a curved surface) is progressively transformed into a profile that ordinarily occurs without highlights or lowlights (e.g., a shadow penumbra generated by the sun). Fig. 2 shows a simple method of testing this prediction. As can be seen in Fig. 2, when a square occluder is oriented such that one of its sides is aligned parallel to the

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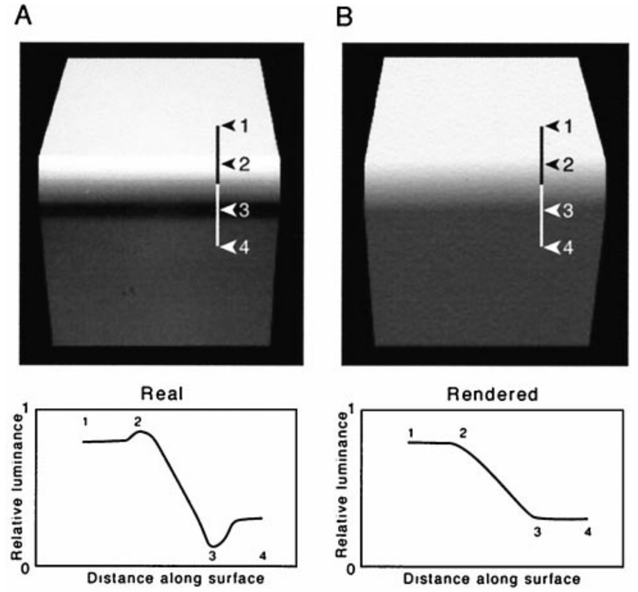


FIG. 1. Similarity of photometric maxima and minima generated by a curved surface and illusory Mach bands. (*A*) Digital photograph of a real-world cube manifesting a photometric highlight and lowlight (as indicated in the luminance profile beneath the photo). (*B*) A computer-generated image of a similar object, but lacking the highlight and lowlight. Despite the objective absence of these adornments, brightness maxima and minima (Mach bands) are apparent in the positions of their photometric counterparts in *A*.

edge of the shadow-casting object, prominent Mach bands are experienced (Fig. 2*A*). When, however, the occluder is rotated away from this position, the illusion is diminished, being least when the diagonal of the occluder is orthogonal to the edge of the shadow-casting object (Fig. 2*B* and *C*).

The basis of the effect experienced in viewing Fig. 2 is that when an edge of the square occluder is parallel to the shadow-casting edge, the linear luminance gradient produced is similar to the gradient produced by a convexly curved surface, although lacking the photometric highlight and lowlight that usually adorn such stimuli. Conversely, when the occluder is rotated such that one of its corners is directed towards the axis of the shadow-casting edge, the luminance gradient produced is similar to the sigmoidal profile of a cast penumbra, which normally lacks these photometric adornments (Fig. 2 and *Appendix*). Indeed, in addition to Mach bands, a curved surface coming out of the plane of the paper is typically seen in Fig. 2*A*, but not in Fig. 2*B* or *C*, consistent with the probable sources of these different gradients.

The Perception of Mach Bands Elicited by the Same Luminance Gradient Is Changed by Altering Information About the Most Probable Source of the Stimulus. A final prediction is that the perception of Mach bands in response to the same luminance gradient should vary as a function of information in the stimulus that makes it more or less likely that the gradient represents an object adorned by highlights and lowlights (i.e., information that alters the probability of the source being a curved or a flat surface). Fig. 3 examines this prediction, using texture, perspective, and shadowing to enhance the probability that the source of the gradient in the upper part of the figure is on a curved surface and that the

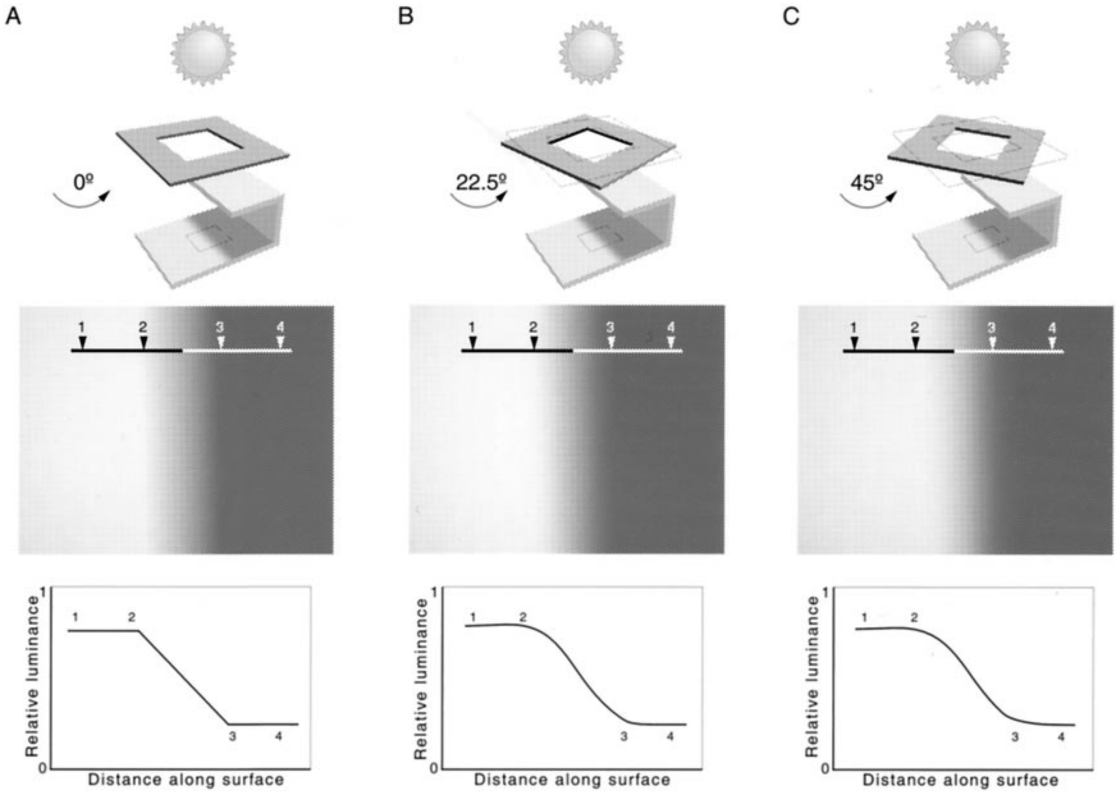


FIG. 2. Salience of Mach bands as a function of the relative linearity of the luminance gradient stimulus [generated here by a square occluder interposed between the light source and the surface on which the shadow is cast, as indicated (*Top*); see *Methods* for details]. (*A*) Linear luminance gradient generated when the leading edge of the square occluder (*Top*) is oriented parallel to the edge of the shadow-casting object. (*B*) Ambiguous gradient created when the occluder is partially turned. (*C*) Sigmoidal gradient created when the occluder is turned 45°. The salience of the Mach band illusion in the digital photographs of the gradients created in this way (*Middle*) is greatest when the penumbral gradient is linear, and least when it is sigmoidal. (*Bottom*) The photometric gradients measured for each of these stimuli. The geometrical rationale for this demonstration is given in the *Appendix*.

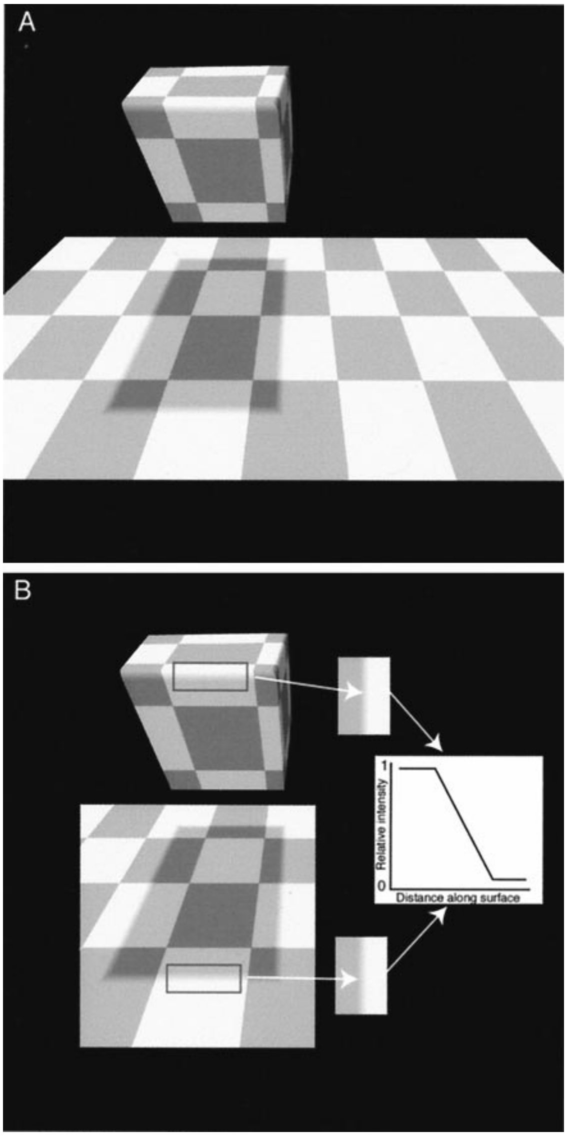


FIG. 3. The perception of Mach bands elicited by a given gradient is changed by information in the stimulus that alters the probability of the source being a curved or a flat surface. (*A*) Depiction of a luminance gradient in two different contexts. Using texture, shadow, and perspective, the gradient in the upper portion of the figure is depicted as arising from a curved surface; the same gradient in the lower part of the figure is depicted as the penumbra of a shadow cast on a flat surface. (*B*) Diagram indicating location of the gradients tested. As indicated in Table 1, subjects invariably perceived the Mach bands associated with the curved surface to be more salient than the bands associated with the penumbra of the shadow cast on the flat surface.

Table 1. Perceptual responses to Mach bands

Subject	Curve	Salience	Penumbra	Salience
1	Yes	10	Yes	5
2	Yes	10	Yes	5
3	Yes	10	Yes	2
4	Yes	10	Yes	3
5	Yes	10	Yes	9
6	Yes	10	Yes	2.5
7	Yes	10	Yes	2.5
8	Yes	10	No	1
9	Yes	10	Yes	2
10	Yes	10	Yes	1.5
11	Yes	10	Yes	2
12	Yes	10	Yes	2
13	Yes	10	Yes	5.5
14	Yes	10	Yes	9
15	Yes	10	Yes	3
16	Yes	10	Yes	5
17	Yes	10	No	1
18	Yes	10	Yes	6
19	Yes	10	Yes	2.5
20	Yes	10	No	1
21	Yes	10	No	1
22	Yes	10	Yes	5
23	Yes	10	Yes	2
Average relative salience		10		3.4

Perceptual responses of 23 naive subjects to the graphic in Fig. 3. Subjects were first shown a standard stimulus (similar to Fig. 4*D*) to familiarize them with Mach bands. They were then shown an enlarged, high quality version Fig. 3*A*, instructed about the two gradients to be compared, and asked to judge whether Mach bands were seen in both locations and whether they were equally salient. If subjects reported that the bands were not of equal strength (as all did), they were then asked to rank the quality of the less salient Mach bands on a scale of 1–10, with 1 defined as imperceptible and 10 defined as being equal to the more salient Mach bands.

source of the identical gradient in the lower part of the scene is the penumbra of a shadow cast on a flat surface. When the perception of the differently depicted gradient was tested in 23 individuals naive about Mach bands and their possible significance, all subjects reported that the illusory bands were more salient (by an average factor of about 3) across the curved surface than across the same gradient depicted as a shadow penumbra on a flat surface (Table 1).

DISCUSSION

In a recent study of simultaneous brightness contrast (2, 3), we concluded that illusions based on the juxtaposition of contrasting territories are the manifestations of empirically driven neural associations (percepts) determined by the relative probabilities of the possible real-world sources of the light returned to the eye rather than the actual qualities of the object or the properties of the stimulus *per se*. The argument for this statistical strategy of brightness perception is summarized in Fig. 4*A*. In brief, the luminance profile in Fig. 4*A* is a conventional stimulus for eliciting a simultaneous contrast illusion: the gray diamond on the dark square appears lighter than the diamond on the lighter square, even though the two test diamonds are equiluminant. In fact, the "scene" in Fig. 4*A* is profoundly ambiguous: the luminance profile could be an evenly illuminated card with different reflectance properties (Fig. 4*B*) or a card having the same surface properties but differently illuminated (Fig. 4*C*) (note that by ambiguity we simply mean a stimulus that has more than one potential real-world source). We therefore proposed that the visual system uses past experience to elicit percepts constructed on the basis of the relative likelihood of the various sources of the stimulus rather than the objective qualities of the stimulus or its retinal effects (recall that a common interpretation of such illusions is that they arise from a direct perception of distorted lower order computations of local or global contrast engendered by lateral interactions). Since the profile in Fig. 4*A* would only sometimes have represented an evenly illuminated surface (the only circumstance in which the two equiluminant test patches would, in reality, have returned the same amount of light to the eye), the perceptual association elicited by the scene is elaborated empirically according to the relative probabilities of the continuum of possible sources underlying the stimulus.

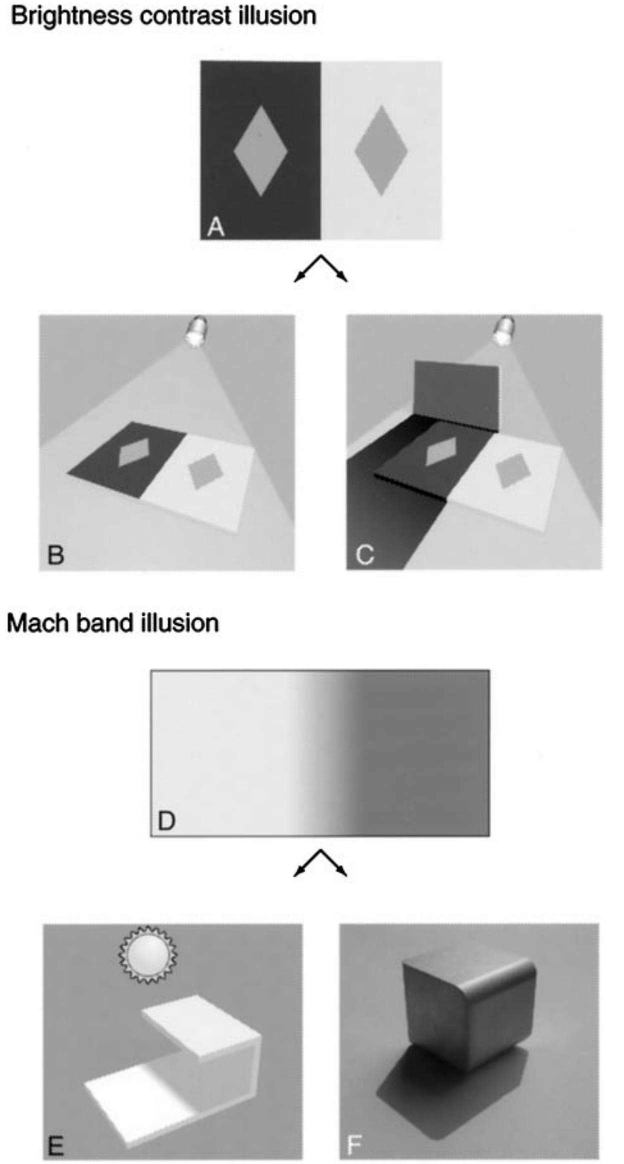


FIG. 4. Mach bands explained in the same framework used to rationalize simultaneous brightness contrast illusions. (*Upper*) When two equiluminant patches (the diamonds) are presented on dark and light backgrounds, respectively, the diamond on the dark background appears lighter than the diamond on the light background (*A*). This standard stimulus for eliciting illusions of simultaneous brightness contrast is ambiguous: it could represent an evenly illuminated card with different surface qualities accounting for the dark and light surrounds of the equiluminant test diamonds (*B*), or a card with uniform surface properties, half of which is shadowed (*C*) (among other possibilities). Because the underlying sources of the stimuli in *B* and *C* are different, they will require different visually guided behaviors: to respond appropriately, the visual system must therefore determine the significance of the stimulus in *A* from the limited information available. Since there is no way to compute the "right answer" (i.e., the actual source of the stimulus) based on a logical principle that could be expressed algorithmically, we have proposed that the visual system generates percepts empirically, the stimulus in *A* eliciting an association (the percept) according to what the stimulus has most often turned out to be. (*Lower*) We now propose the same explanation for Mach bands, which are elicited by luminance gradients rather than luminance boundaries. Like the stimulus in *A*, the luminance gradient in *D* is ambiguous: as demonstrated in earlier figures, the profile could be the penumbra of a cast shadow (*E*) or the gradient generated by a curved surface (*F*). The penumbral gradients of cast shadows lack photometric highlights and lowlights, whereas the gradients generated by curved surfaces typically have luminance maxima and minima. We take the Mach bands elicited by the ambiguous stimulus in *D* to be an association engendered empirically by past experience (see text).

The biological rationale of this strategy is that by triggering associations weighted in this way by past experience (presumably both phylogenetic and ontogenetic), ambiguous visual stimuli will always generate percepts that have the greatest likelihood of representing the stimulus for what it has most often turned out to be. This empirical strategy of experiencing visual stimuli allows the observer to produce an optimal behavioral response to the ambiguous luminance profile in Fig. 4*A*, or indeed to any luminance profile.

Our explanation of Mach bands simply carries this theory of vision into a related domain of brightness perception in which the stimuli are luminance gradients rather than contrasting territories. For instance, the luminance gradient in Fig. 4*D* is, like the stimulus in Fig. 4*A*, ambiguous: this standard stimulus for the Mach band illusion (see Figures 1 and 3 in ref. 1) could represent a shadow cast on a flat surface (Fig. 4*E*) or a gradient preceding an attached shadow on a curved surface (Fig. 4*E*) (as well as a painted surface, which is the source of the stimulus in the laboratory). According to our hypothesis, the association elicited in response to any such gradient is a construct based on the relative probabilities of the possible real-world sources of the stimulus (determined by the observer's and the species' past experience with similar stimuli whose significance will have been ascertained empirically by whether the ensuing visually guided behavior was successful). In the case of the stimulus in Fig. 4*D*, the features of this linear luminance profile—like the features of Mach's spinning disk—will often have been adorned by highlights and lowlights; consequently, the stimulus triggers an association that incorporates these features according to the relative probability of a source adorned in this way.

In support of this interpretation, when the stimulus is made more like an ordinary penumbra (by making the luminance gradient increasingly sigmoidal), the perception of Mach bands is diminished (see Fig. 2). This diminishment presumably occurs because the altered stimulus, for the same statistical reasons, now triggers an association that reflects the increased likelihood of a source that lacks highlights and lowlights. Conversely, when a luminance gradient is made more linear, the Mach-band illusion is enhanced, in this case because the association induced by the stimulus is now influenced by the greater probability that a linear gradient will be adorned with a photometric highlight and lowlight. The ability to alter the salience of Mach bands elicited by the same luminance gradient through manipulating information about the provenance of the gradient (see Fig. 3) similarly supports this interpretation.

Finally, we note that the conventional explanation of Mach bands as perceptions arising directly from lateral interactions among the retinal ganglion cells or other lower order visual neurons (see refs. 4 and 5 for extensive reviews) is undermined by the fact that the perception of the illusory bands is diminished or absent when subjects view a step change in luminance between two adjacent surfaces (i.e., an edge) (see, for example, refs. 6 and 7). If reciprocal lateral interactions were at the root of the illusion, Mach bands should be most salient when viewing luminance boundaries between two surfaces, not luminance gradients. In contrast, the concept of Mach bands as the result of an empirical strategy of vision correctly predicts the diminished salience of the illusion in response to step gradients (because such edges are not, in reality, typically adorned by highlights and lowlights).

Conclusion. Luminance gradients that elicit the perception of Mach bands are, by virtue of the physical properties of reflected light and the prevalence of curved surfaces in the environment, frequently adorned with photometric highlights and lowlights, a statistical conjunction that we take to be the source of this illusion. The facts that support this conclusion are: (*i*) stimuli of the sort used by Mach and others to elicit this illusion in the laboratory are unusual, whereas similar luminance gradients adorned by highlights and lowlights are commonplace in natural settings; (*ii*) the appearance of Mach bands accords with the appearance of real-world highlights and lowlights; (*iii*) the perceptual prominence of Mach bands can be modulated by altering the configuration of the stimulus gradient to make the stimulus either more or less consistent with a real-world source that is normally adorned with highlights and lowlights; (*iv*) the perception of Mach bands elicited by the same luminance gradient can be altered by contextual information that indicates whether the gradient lies on a curved or flat surface; and (*v*) Mach bands are most strongly perceived in response to luminance gradients and least prominent at a step edge between two surfaces. The neuronal basis of the percepts triggered by the luminance gradients we have examined is presumably patterns of synaptic connectivity engendered by both phylogenetic and ontogenetic experience.

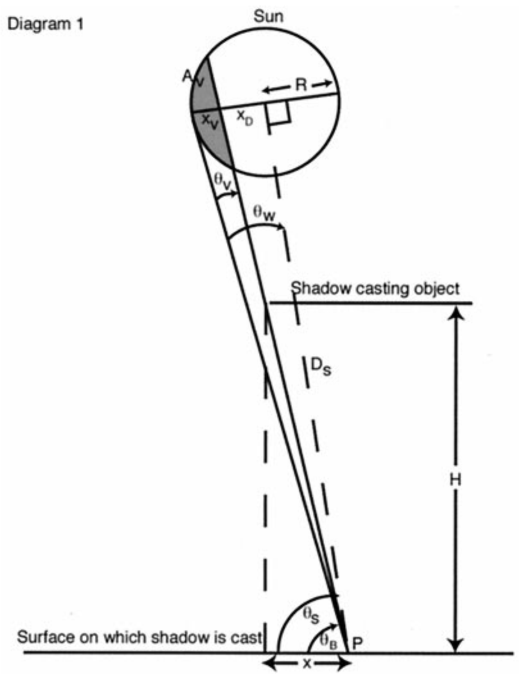
The behavior of Mach bands appears in all respects similar to the genesis of simultaneous brightness contrast illusions (2, 3). In fact, such distorted percepts of luminance—Mach bands now included—are no more or less illusory that any other visual percepts, all of which we take to signify the probabilistic operation of the visual system as it disambiguates scenes according to a fundamentally empirical strategy.

Appendix

Computation of the Luminance Gradients Associated with the Penumbras of Shadows in Sunlight. The amount of light (L) reaching any point (P) on the surface on which a shadow and its penumbra are cast can be determined by multiplying the amount of light (L_F) reaching any fully illuminated point by the ratio of the exposed area of the sun's disk (A_V) to the total area of the disk (A_T). Thus,

$$L_P = \frac{A_V}{A_T} \cdot L_F \qquad [1]$$

Diagram 1



Determining the solution of Eq. **1** entails two subsidiary problems: (*i*) finding the length of line (X_V) from the perimeter of the sun's disk to the edge of the exposed portion to the disk as seen from point P (see Diagram 1) and (*ii*) using X_V, determining the exposed area of the sun's disk at P (that is, determining A_V).

With respect to determining X_V,

$$X_V = R - X_D \qquad [2]$$

and

$$X_D = D_S \tan(\theta_W - \theta_V). \qquad [3]$$

When

$$\theta_V = \tan^{-1}\left(\frac{H}{X}\right) - \theta_B \qquad [4]$$

and

$$\theta_B = \theta_S - \theta_W, \qquad [5]$$

$$X_V = D_S \tan(\theta_W) - D_S \tan\left(\theta_W - \left(\tan^{-1}\left(\frac{H}{X}\right) + \theta_W - \theta_S\right)\right) \qquad [6]$$

and

$$X_V = D_S \tan(\theta_W) - D_S \tan\left(\theta_S - \tan^{-1}\left(\frac{H}{X}\right)\right). \qquad [7]$$

With respect to determining A_V,

$$A_V = \int \sqrt{R^2 - x^2 d_x}, \qquad [8]$$

$$A_V = \frac{x}{2}\sqrt{R^2 - x^2} + \frac{R^2}{2}\sin^{-1}\frac{x}{R}, \qquad [9]$$

and

$$A_V = \left(\frac{x - R}{2}\sqrt{R^2 - (x - R)^2} + \frac{R^2}{2}\sin^{-1}\frac{x - R}{R}\right). \qquad [10]$$

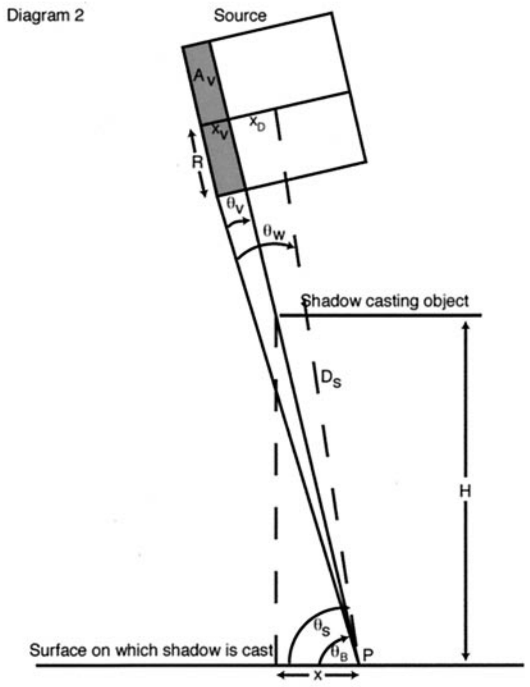
To shift the integral's starting point to the edge of the sun's disk rather than the center (see Diagram 1), (X − R) is substituted for X. Thus

$$A_V = \left[(x - R)\sqrt{2Rx_V - x_V^2} + R^2 \sin^{-1}\left(\frac{x_V - R}{R}\right)\right]_0^{x_V}, \qquad [11]$$

$$A_V = (x_V - R)\sqrt{2Rx_V - x_V^2} + R^2 \sin^{-1}\left(\frac{x_V - R}{R}\right) + \frac{\pi R^2}{2}, \qquad [12]$$

and

$$A_V = -D_S \tan\left(\theta_S - \tan^{-1}\left(\frac{H}{x}\right)\right) \times \sqrt{2D_S^2 \tan(\theta_W) - 2D_S^2 \tan\left(\theta_S - \tan^{-1}\left(\frac{H}{x}\right)\right)\tan\theta_W - \left(D_S \tan\left(\theta_S - \tan^{-1}\left(\frac{H}{x}\right)\right)\right)^2} + D_S^2 \tan^2(\theta_W)\sin^{-1}\left(\frac{-D_S \tan\left(\theta_S - \tan^{-1}\left(\frac{H}{x}\right)\right)}{D_S \tan(\theta_W)} + \frac{\pi D_S^2 \tan^2(\theta_W)}{2}\right) \quad [13]$$



Returning to Eq. **1**, the luminance profile across a penumbra is calculated by solving L_P for all values of X (the distance across the surface on which the shadow is cast; see Diagram 1). Note that the total area of the sun's disk is given by

$$A_T = \pi R^2, \quad [14]$$

$$A_T = \pi(D_S^2 \tan^2(\theta_W)), \quad [15]$$

and

$$A_T = \pi D_S^2 \tan^2(\theta_W)A. \quad [16]$$

When the shape of the light source casting a shadow is rectilinear instead of circular, the determination of X_V and A_V differs accordingly.

With respect to determining X_V,

$$R = D_S \tan(\theta_W), \quad [17]$$

$$X_V = D_S \tan(\theta_W) - D_S \tan\left(\theta_S - \tan^{-1}\left(\frac{H}{x}\right)\right). \quad [18]$$

With respect to determining A_V,

$$A_V = x_V \cdot 2R, \quad [19]$$

$$A_V = 2Rx_V, \quad [20]$$

and

$$A_V = 2D_S^2 \tan^2(\theta_W) - 2D_S^2 \tan(\theta_W)\tan\left(\theta_S - \tan^{-1}\left(\frac{H}{x}\right)\right). \quad [21]$$

The modified meaning of the terms in these equations are indicated in Diagram 2. Finally, note that the formula for the total area of rectangular light source is given by

$$A_T = (2R)^2, \quad [22]$$

$$A_T = 4R^2, \quad [23]$$

and

$$A_T = 4D_S^2 \tan^2(\theta_W). \quad [24]$$

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